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DIRECTION OF A LINE

J. B. DAVIS
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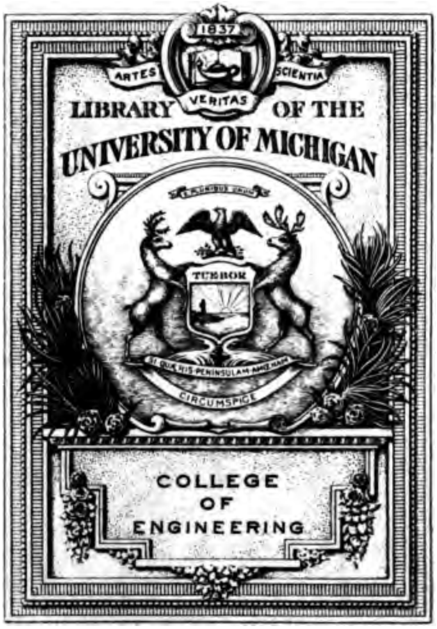

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DIRECTION OF A LINE

NEEDLE COMPASS WORK

1. For needle compass work the following is sufficient:

On Plate I, B is the Pole Star, *α* Ursae Minoris, mag. 2.3; A is δ Ursae Minoris, mag. 4.7, next to the Pole Star in the handle of the little dipper; C is 51 Cephei, mag. 5.2; and D is 32 Camelopardalis, mag. 5.6.

The Pole is a little less than one-third of the distance from the intersection of A C and B D to A B. *a d* is parallel to A D.

D E and F H, the diagonals of the quadrilateral D F E H, intersect close to the Pole.

The line I L passes near the Pole.

A fourth line passing near the Pole is also shown.

The names and magnitudes of the stars are given so they can be learned from the map of the northern stars, Plate II.

By means of these various lines the direction of the Pole can be taken, probably within a manute of arc, at any time when these stars are visible, and all the necessary work of observation done in half an hour.

2. Set a light over a stake. At a convenient distance south of the light fix the position of another stake by holding a plumb line in range with the light and the pole. A good way of doing this is to drive two tall stakes in the ground south from the lantern and nail a board horizontally to them so that a person when seated can look under the board and view the stars to be used, especially A, B, C, D. Hold the string of a plumb bob against the south side of the board while bringing it into range with the light and the pole. After the range is obtained mark on the board the place of the string. Repeat these trials, marking each position of the string. Place the marks on the board at different elevations to prevent one mark covering another. Select the average position of these marks and notch the board at this place. Transfer the position of the notch to the ground with the plumb bob. Drive a stake here. Drive a nail in the stake directly beneath the notch.

For something more precise one may use the process of one of the methods which follow. The observations may be made by means of a long plumb line held taut by a large, heavy weight suspended in water. South of this plumb line drive two stakes and nail a board on top of them leveled both ways. This board should be about four feet from the ground and stand E. and W. Attach a compass sight to a short board so it stands square to the bottom of the board. Observe a star by keeping the compass sight in range with it and the plumb line as it approaches its elongation. When it reaches its elongation the star will appear to pass directly along the plumb line and the slit in the compass sight. Keep the compass sight at the place it is in when the star is at its elongation. Transfer the position of the slit in the compass sight to a stake marked for the star and driven into the ground, marking the exact place by a nail. Another stake, marked for the star, may be set beyond the plumb line with the help of a lantern. The true bearing of this line will be the azimuth of the star at its elongation. This may be computed if the latitude is known, or two stars may be observed and the lines to them marked as above noted. Afterwards, a meridian line may be laid out by the last method given herein,—Sec. 80.

3. Deviation of the Needle.

Given the direction of a line, that is its true bearing, or azimuth.

Set the ring on the compass plate so the needle will read the true bearing of the line when the compass is sighted along it. The reading of the vernier to the compass ring will be the amount of the deviation of the needle. The direction of it may be observed by setting up the compass so the north end of the needle will stand at the N. zero on the compass ring, and noting which way the north end of the needle would stand from the N. zero of the compass ring if the ring should be turned so its vernier would read zero. If the north end of the needle would then stand east of the N. zero of the ring, the deviation is East; if west, the deviation is West.

If the compass has a fixed ring and no deviation vernier, take the bearing of the line with the needle.

Find the traverse angle of the line, the needle being the reference line N S, by means of the directions on the diagram below.

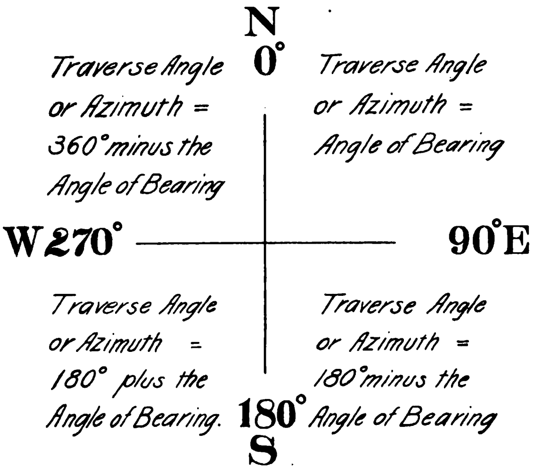
N
0°
Traverse Angle or Azimuth = 360° minus the Angle of Bearing
Traverse Angle or Azimuth = Angle of Bearing
W 270°
90° E
Traverse Angle or Azimuth = 180° plus the Angle of Bearing.
Traverse Angle or Azimuth = 180° minus the Angle of Bearing
180°
S

Fig. 1.

Find the azimuth of the line from its true bearing, using the same diagram.

Subtract the traverse angle of the line, found from its needle bearing, from the azimuth of the line, found from its true bearing.

The result will be the azimuth of the needle, or its deviation counted in right handed rotation.

The corresponding bearing, if desired, may be found by reference to the above diagram.

This last method will apply even though the north end of the needle should point south east, or south west, as is the case in some parts of the western hemisphere.

4. After a term of years a new plat of the stars on Plate I should be made, as their relative positions change somewhat. This plate is a radial projection of the stars upon a plane tangent to the celestial sphere at its north pole. The distance from the pole to the projection of each star (to the scale used) is the tangent of its polar distance, or the cotangent of its declination.

TRIGONOMETRY

5. "**Trigonometry** is that branch of mathematics which treats of methods of subjecting angles and triangles to numerical computation."—Wm. Chauvenet.

This includes angular analysis.

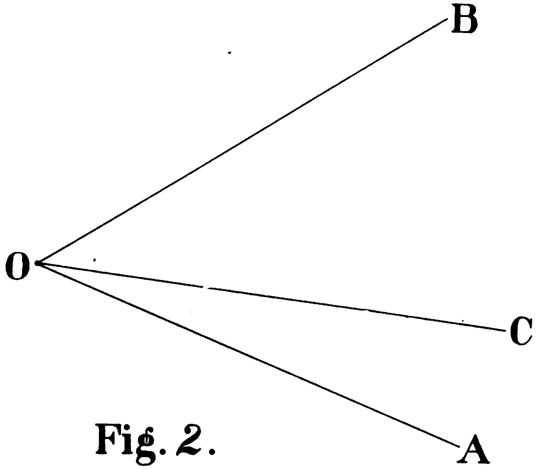

Fig. 2.

6. **Plane Trigonometry** is the mathematics of the relative positions of points in a plane, both as to direction and distance; and the relations of plane angles in general, which is the Angular Analysis.

7. **Spherical Trigonometry** is the mathematics of the relative directions of points in space. Three lines O A, O B, and O C, diverging from a point O, are sufficient for this purpose.

This, then, is the mathematics of the triedral angle O — A B C.

A = the angle between the planes C O A and A O B.

B = the angle between the planes A O B and B O C.

C = the angle between the planes B O C and C O A.
a = the plane angle B O C, opposite A.
b = the plane angle C O A, opposite B.
c = the plane angle A O B, opposite C.

Any three of these six quantities being given the others may be computed. Their relations may be found by the principles of Geometry and Plane Trigonometry, as follows:

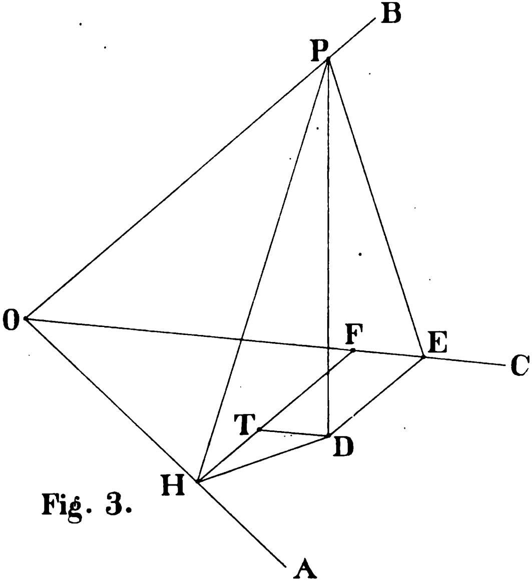

Fig. 3.

P is any point on O B.
Plane P H D is perpendicular to O A.
Plane P E D is perpendicular to O C.
P D is perpendicular to plane A O C.
P H and H D are perpendicular to O A.
P E and E D are perpendicular to O C.
H F is parallel to E D.
D T is parallel to O C.

Triangle P O E is right angled at E. $POE = a$.
Triangle H O F is right angled at F. $HOF = b$.
Triangle P O H is right angled at H. $POH = c$.
Triangle P H D is right angled at D. $PHD = A$.
Triangle P E D is right angled at D. $PED = C$.
Triangle D H T is right angled at T. $DHT = b$.

$$PE = OP \sin a.$$
$$PH = OP \sin c.$$
$$PD = PE \sin C.$$
$$PD = PH \sin A.$$

$$\sin c \sin A = \sin a \sin C.$$

As P may be assumed on either O A or O C there are two other equations similar to this, completing the group below.

$$\begin{aligned} \sin a \sin B &= \sin b \sin A. \\ \sin b \sin C &= \sin c \sin B. \qquad (1) \\ \sin c \sin A &= \sin a \sin C. \end{aligned}$$

$$OE = OP \cos a.$$
$$OF = OH \cos b.$$
$$OH = OP \cos c.$$
$$TD = HD \sin b.$$
$$HD = PH \cos A.$$
$$OE = OF + TD.$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

As P may be assumed on either O A or O C, there are two other equations similar to this, completing the group below.

$$\begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c \cos A. \\ \cos b &= \cos c \cos a + \sin c \sin a \cos B. \qquad (2) \\ \cos c &= \cos a \cos b + \sin a \sin b \cos C. \end{aligned}$$

$$\begin{aligned} \mathrm{H\,F} &= \mathrm{O\,H} \sin b. \\ \mathrm{H\,T} &= \mathrm{H\,D} \cos b. \\ \mathrm{D\,E} &= \mathrm{P\,E} \cos C. \\ \mathrm{D\,E} &= \mathrm{H\,F} - \mathrm{H\,T}. \end{aligned}$$

$$\sin a \cos C = \sin b \cos c - \cos b \sin c \cos A.$$

As P may be assumed on either O A or O C, there are two other equations similar to this, completing the group below.

$$\begin{aligned} \sin a \cos C &= \sin b \cos c - \cos b \sin c \cos A. \\ \sin b \cos A &= \sin c \cos a - \cos c \sin a \cos B. \\ \sin c \cos B &= \sin a \cos b - \cos a \sin b \cos C. \end{aligned} \tag{3}$$

If B and C be interchanged, and therefore b and c, this group becomes

$$\begin{aligned} \sin a \cos B &= \sin c \cos b - \cos c \sin b \cos A. \\ \sin b \cos C &= \sin a \cos c - \cos a \sin c \cos B. \\ \sin c \cos A &= \sin b \cos a - \cos b \sin a \cos C. \end{aligned} \tag{4}$$

If at O three lines be drawn perpendicular to the faces of the triedral O — A B C, —

O A′ perpendicular to the plane B O C,
O B′ perpendicular to the plane C O A, and
O C′ perpendicular to the plane A O B, these lines will define a triedral O — A′ B′ C′ on the opposite side of O from O — A B C. These triedrals are *supplementary*.

O A is perpendicular to the plane B′ O C′.
O B is perpendicular to the plane C′ O A′.
O C is perpendicular to the plane A′ O B′.

Using the same notation for O — A′ B′ C′ as for O — A B C.

$A' =$ the diedral whose edge is $O\,A' = 180^\circ - a$.
$B' =$ the diedral whose edge is $O\,B' = 180^\circ - b$.
$C' =$ the diedral whose edge is $O\,C' = 180^\circ - c$.
$a' =$ the plane face angle $B'\,O\,C' = 180^\circ - A$.
$b' =$ the plane face angle $C'\,O\,A' = 180^\circ - B$.
$c' =$ the plane face angle $A'\,O\,B' = 180^\circ - C$.

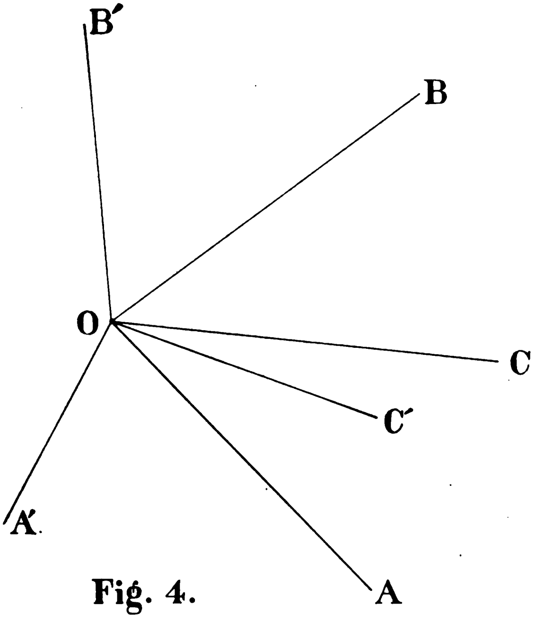

Fig. 4.

The same forms apply to one triedral as to the other, so

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'.$$

If in this equation the above equivalents be substituted

$$\cos A = - \cos B \cos C + \sin B \sin C \cos a.$$

The group is

$$(5)\quad \begin{aligned} \cos A &= - \cos B \cos C + \sin B \sin C \cos a. \\ \cos B &= - \cos C \cos A + \sin C \sin A \cos b. \\ \cos C &= - \cos A \cos B + \sin A \sin B \cos c. \end{aligned}$$

These equations show that when A, B, and C are known, a, b, and c can be found. This is not the case in plane triangles.

Groups (3) and (4) may be treated in the same manner and groups (6) and (7) derived.

$$\begin{aligned}&\sin A \cos b = \sin C \cos B + \cos C \sin B \cos a.\\(6)\qquad&\sin B \cos c = \sin A \cos C + \cos A \sin C \cos b.\\&\sin C \cos a = \sin B \cos A + \cos B \sin A \cos c.\end{aligned}$$

$$\begin{aligned}&\sin A \cos c = \sin B \cos C + \cos B \sin C \cos a.\\(7)\qquad&\sin B \cos a = \sin C \cos A + \cos C \sin A \cos b.\\&\sin C \cos b = \sin A \cos B + \cos A \sin B \cos c.\end{aligned}$$

By dividing the equations in groups (3) and (4) by those in group (1) after writing them in the form

$$\frac{\sin a \sin B}{\sin A} = \sin b$$

those in groups (8) and (9) may be obtained. No additional relations are developed by other transformations.

$$\begin{aligned}&\sin A \cot B = \sin c \cot b - \cos c \cos A.\\(8)\qquad&\sin B \cot C = \sin a \cot c - \cos a \cos B.\\&\sin C \cot A = \sin b \cot a - \cos b \cos C.\end{aligned}$$

$$\begin{aligned}&\sin A \cot C = \sin b \cot c - \cos b \cos A.\\(9)\qquad&\sin B \cot A = \sin c \cot a - \cos c \cos B.\\&\sin C \cot B = \sin a \cot b - \cos a \cos C.\end{aligned}$$

8. These nine groups are sufficient for a theoretical solution of every case of the relative positions of points in space. It is sometimes said that the first equations in groups (1), (2), and (3) are sufficient for solving every case in Spherical Trigonometry. This means that from these three basic equations all of the others may be derived, as has been outlined above. From these groups special equations are prepared for convenient calculation, the number of which is large.

9. **The Sphere.** If P be assumed on all of the six lines O A, O B, O C, O A′, O B′ and O C′, at the same distance from O, these six points will be situated on the surface of a sphere with its center at O, and radious O P. The planes of all six faces of the triedrals will be great circles of this sphere. Their intersections with the surface of the sphere will be arcs of great circles joining the points on the six lines and forming two spherical triangles, which may be designated as A B C and A′ B′ C′. In the notation used above A, B, C, and A', B', C', are the angles of these triangles, and a, b, c, and a', b', c', are their **sides,** so called. From this arises the term Spherical Trigonometry. The two triangles A B C, and A′ B′ C′, **polar triangles,** that is, each vertex of either triangle is the pole, on the surface of the sphere, of the **side** of the other triangle bearing the same letter, as A is the pole of a' and B′ is the pole of b.

10. If C be made 90° the equations for **Spherical Right Triangles** may be derived from the groups above. The simplest of these and the corresponding equations for Plane Right Triangles are shown in the groups below. This may be of assistance in remembering them.

In Plain Right Triangles		In Spherical Right Triangles	
$\sin A = \frac{a}{c}$	$\sin B = \frac{b}{c}$	$\sin A = \frac{\sin a}{\sin c}$	$\sin B = \frac{\sin b}{\sin c}$
$\cos A = \frac{b}{c}$	$\cos B = \frac{a}{c}$	$\cos A = \frac{\tan b}{\tan c}$	$\cos B = \frac{\tan a}{\tan c}$
$\tan A = \frac{a}{b}$	$\tan B = \frac{b}{a}$	$\tan A = \frac{\tan a}{\sin b}$	$\tan B = \frac{\tan b}{\sin a}$
$\sin A = \cos B$	$\cos B = \sin A$	$\sin A = \frac{\cos B}{\cos b}$	$\sin B = \frac{\cos A}{\cos a}$
$c^2 = a^2 + b^2$		$\cos c = \cos a \cos b$	
$1 = \cot A \cot B$		$\cos c = \cot A \cot B$	

11. In using any of the equations, in any of the groups, for computation the results should invariably be checked by repeating the computation by other equations, or by the formation of check equations for this

purpose, unless the work itself to which the computation applies, furnishes independent checks. The independent checks are preferable, almost always.

12. Further knowledge of Trigonometry may be obtained from "Plane and Spherical Trigonometry," by William Chauvenet. In his preface the author says, "I have in this treatise endeavored to arrange a course of trigonometrical study sufficiently extensive to enable the student to comprehend readily any applications of trigonometry he may meet with in the works of the best modern mathematicians. With this object, some topics have been introduced which are not usually found in works devoted specially to this subject." This was in 1850.

ASTRONOMY

13. Some knowledge of astronomical terms in common use is necessary for finding the direction of a line by the more precise methods. In speaking of any celestial object, as the sun, or a star, its altitude and direction are used to define its position. These terms may be used with more precision for purposes of instrumental observation. The value of the angle of altitude from the horizon, and the angle from the meridian around to the vertical plane of the object, as a star, are used. The latter angle is called Azimuth and may be considered as reckoned in the plane of the horizon. It is commonly reckoned from the north point of the horizon, either to the east, or to the west, but in some work it is reckoned from the south. Thus it is that the azimuth and altitude of an object define its position,—really its specific direction from the observer. The planes of reference are the meridian of the observer P Z P′, Fig. 5, and his horizon, represented by the circle whose center is C, in Fig. 5. If S be a star, its azimuth from the north point of the horizon, H, is H C H″, and its altitude is H″ C S,—this latter angle being projected upon the plane of the horizon in a straight line. Azimuth and altitude are one system of coordinates by means of which the position of a star, S, may be defined. There are a number of others in common use, having other planes of reference. Some of these are illustrated in Figures 5, 6, and 7, where the same letter, or symbol, designates the same thing in all of them. As these systems of coordinates are all reckoned in angular measure and involve the solution of problems in Spherical Trigonometry, and as celestial objects, such as the sun, moon, planets, and stars, are referred to an indefinitely large sphere, called the Celestial Sphere, which has the same center and axis as the earth; the lines of a

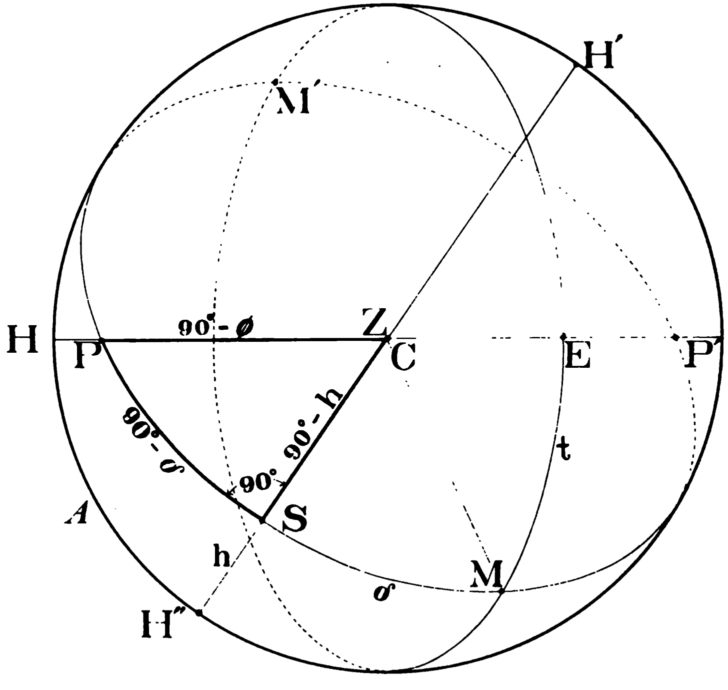

Fig. 5.

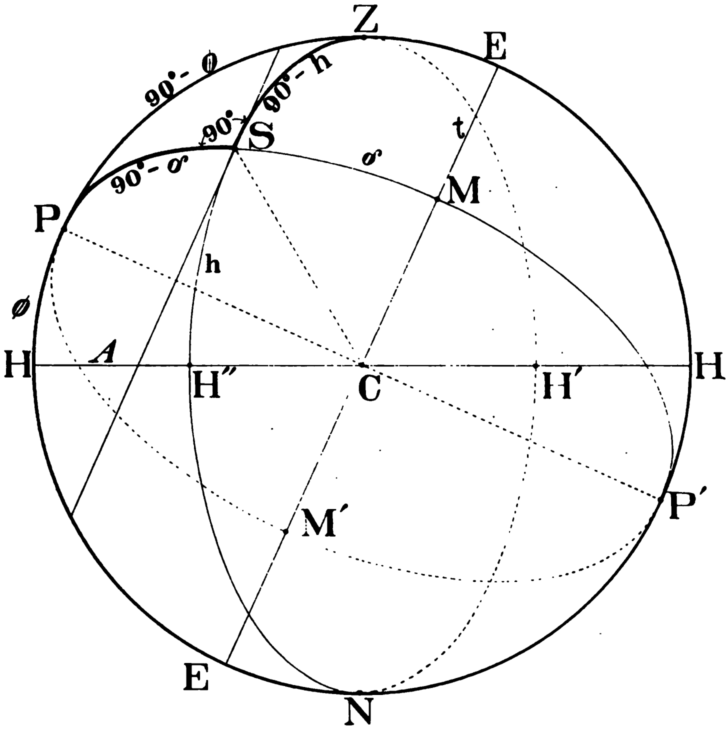

Fig. 6.

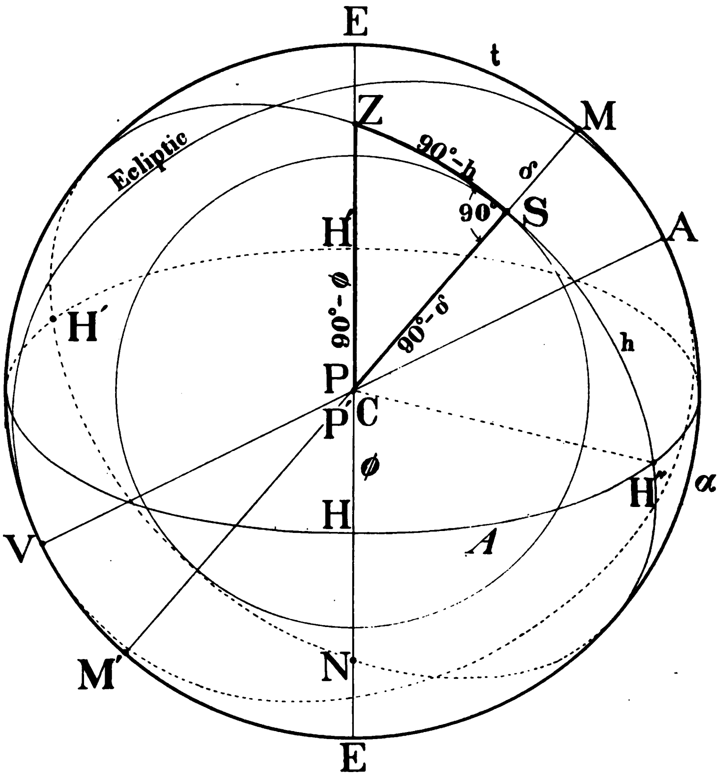

Fig. 7.

sphere are used in these representations. Figure 5 is a projection of the sphere on the plane of the observer's horizon. Figure 6 is a projection of the sphere upon the observer's meridian. Figure 7 is a projection of the sphere upon the plane of the equator. These three different views aid in obtaining a clear idea of the lines of the sphere, as well as of the spherical triangle of position, P Z S. In this triangle P is the pole, Z the zenith of the observer, and S is the star, or other celestial object.

If the axis of the earth be considered as indefinitely extended in both directions it will define the poles P and P′ of the celestial sphere.

If a weight be suspended by a string and be allowed to come to rest, the line of the string indicates the direction of the vertical line, and shows the line of action of the resultant of gravity at the place where the weight is suspended.

If the vertical line be conceived of as being indefinitely extended in both directions, it will define the Zenith, Z, upward, and the Nadir, N, downward.

AZIMUTH AND ALTITUDE

14. **The Horizon** at any place is a plane perpendicular to a vertical line at that place, and extending without limit in all directions. The Astronomical Horizon is a similar parallel plane passing through the center of the earth. In observations upon the so called "Fixed" stars no account need be taken of the distance between these two horizons; its effects are imperceptible. The observer can consider himself as being at the center of the earth, C. In observations upon the sun, moon, and planets, this distance must be considered and a correction made for the parallelax caused by it. Observations of altitude at sea which are referred to the visible horizon must be corrected for the dip of the horizon, due to the elevation of the eye of the observer above the sea, the curvature of the earth, and atmospheric refraction.

15. **The Meridian** of a place is the plane of the vertical line at that place and the axis of the celestial sphere, P Z P′ N, in the figures.

16. **The Meridian and Horizon** of a place are the reference planes for Azimuth, H C H″, and Altitude, H″ C S, above referred to.

17. **A Vertical Plane** at a place is any plane, without limit in its extent, that contains the vertical line at that place.

18. **The Vertical Circle** of a star, S, or other celestial object, is a vertical plane passing through that object, Z S H″ N.

19. **The Azimuth** of S is the angle between its vertical circle and the meridian of the place of observation. This may be reckoned on the horizon, as H C H″.

20. **The Altitude,** H″ C S, of S is the angle between a visual line, C S, to it and the horizon of the place of observation. It is reckoned in the vertical circle of S.

21. **Azimuth and Altitude,** thus reckoned, define the direction of S with accuracy. These constitute the system of coordinates for defining the direction of S with respect to objects on the earth. The engineer, or surveyor, is more directly concerned with the azimuth. With his transit he may observe the angle between the vertical circle of a star and any reference line on the ground which he may choose,—really this is the difference between the azimuth of the line and that of the star. He may compute the azimuth of the star and thence find the azimuth, or direction, of the line.

22. **The Prime Vertical** is the vertical circle perpendicular to the meridian at a place of observation. This is sometimes used as a plane of reference.

LATITUDE AND LONGITUDE

23. **The Equator** is a plane, without limit, passing through the center of the celestial sphere and perpendicular to the axis of daily rotation of the earth. It is the plane of the earth's equator indefinitely extended.

24. **The Latitude** of a place on the earth is its angular distance from the equator, either north or south, reckoned in the meridian of the place.

25. **The Longitude** of a place on the earth is the angular distance of its meridian from some reference meridian, as the Meridian of Greenwich Observatory, England, reckoned on the equator. Time is directly convertible into an angle.

26. **Latitude and Longitude,** thus reckoned, define the position of a place on the earth with accuracy. These constitute the system of coordinates for this purpose.

DECLINATION AND HOUR ANGLE

27. **The Meridian of any celestial object,** S, is a plane without limit, P S M P′ M′, passing through that object and the axis of daily rotation of the earth.

28. **The Declination** of any celestial object, S, is the angle, M C S, between a line, C S, from the center of the celestial sphere to S, and the equator. It is reckoned in the meridian of S and corresponds to latitude on the earth.

29. **The Hour Angle** of any celestial object, S, is the angle M C E between the meridian of S and the meridian of the place of observation. The hour angle may be considered as reckoned on the equator.

30. **The Declination and Hour Angle** of S are its coordinates in another system whose planes of reference are the equator and the meridian of the place of observation.

DECLINATION AND RIGHT ASCENSION

31. **The Declination** is defined in **28.** To define right ascension certain other planes of the celestial sphere must be described.

32. **The Ecliptic** is the plane of the earth's orbit about the sun, extended without limit. This plane intersects the surface of the earth in the line marked Ecliptic on maps and globes, also in Fig. 7. The plane of the ecliptic passes through the center of the earth and is a great circle of the celestial sphere. It intersects the equator in a diameter A V of the sphere. This diameter defines what are called the equinoxes V and A.

33. As the earth revolves about the sun, the sun is made to appear in that part of the firmament opposite the position of the earth itself. During a year the sun appears to pass entirely around the sky, indicating the place of the ecliptic amongst the stars. As the ecliptic intersects the equator in a diameter of the sphere, the sun will appear to be in the equator twice a year, the places being in opposite directions from C, the center of the sphere, and on the diameter of intersection. These places are the equinoxes.

34. **The Vernal Equinox,** V, is the one which the sun appears to pass in the spring about March 21st.

35. **The Autumnal Equinox,** A, is the one which the sun appears to pass in the autumn about September 21st.

36. **The Equinoctial Colure** is the meridian of the equinoxes, A C V, Fig. 7, or the plane of the axis of the sphere and the diameter of the equinoxes.

38. **Right Ascension** is the angle, usually counted in time, between the equinoctial colure and the meridian of a celestial object reckoned from the vernal equinox in the direction of the earth's daily rotation. It is counted

through 360°, or 24 hours. It is commonly reckoned on the equator. In Fig. 7, M′ P S M is the meridian of S, a star or other celestial object, and V P A is the equinoctial colure. The larger circle whose center is C is the equator. The right ascension of S is the convex angle V C S, or, following the arc of the equator, V E A M is the right ascension of S. It corresponds to longitude on the earth.

39. **The Declination and Right Ascension** of S are its coordinates in a system whose planes of reference are the equator and the equinoctial colure.

40. These various systems of coordinates are so connected as to permit transformations from one to another to be made. This is done by means of the equations of Spherical Trigonometry, which are really equations for transformation of coordinates in space. The problems of finding azimuth, or the direction of a line, time, latitude and longitude by observation, are of this general class though seldom presented in this guise.

TIME

41. Astronomers use true, or apparent, solar time; mean solar time; and siderial time.

42. **True, or Apparent Solar Time,** is the hour angle of the Sun, reckoned in time. The day is the interval between two successive lower, or midnight, transits of the Sun over the same meridian. The successive days are not of equal length and ordinary clocks and chronometers cannot be made to keep this time.

43. **Mean Solar Time** is the hour angle (in time) of an imaginary mean sun, regarded as situated in the equator, the successive transits of which over the same meridian occur at equal intervals. Clocks, watches, and navigator's chronometers keep this time. The mean solar day is made to follow the true, or apparent, solar day as closely as possible. A clock keeping mean solar time will

give correct apparent solar time four times a year. The greatest divergence amounts to rather more than one quarter of an hour.

44. **The Equation of Time** given in the Ephemeris and Nautical Almanac is the difference between apparent solar time and mean solar time. It changes sign four times a year.

45. **Siderial Time** is the hour angle (in time) of the vernal equinox. Siderial clocks (so called) keep siderial time and show the right ascension of the meridian for which they are set. A siderial day is the interval between two successive upper transits of the vernal equinox. This day is shorter than the mean solar day by $3^m\ 55.^s909$ of mean solar time. It comprises 24 siderial hours which are counted from 0 to 24. The siderial clock and the mean solar time clock will be together once a year, about March 23rd, as the year of 365.2422 solar days equal 366.2422 siderial days.

46. **The Civil Day** begins at midnight. A. M. is the interval, counted from 0 to 12 hours, from the lower transit of the mean sun till its upper, or noon, transit. P. M. is the succeeding like interval, counted from 0 to 12 hours.

47. **The Astronomical Day** begins at noon of the civil day of the same date and is divided into 24 hours counted from 0 to 24.

48. These three methods of reckoning time are convertible, that is, if the time is given by either method, the corresponding time by the others can be found.

49. For more information upon the subject of Time see the American Ephemeris and Nautical Almanac.

ADDITIONAL TERMS

50. **The Latitude** of a place is the declination of its zenith,—also the altitude of the pole.

51. **The Polar Distance** of a star is 90° minus its declination.

52. **Circumpolar Stars** are those whose polar distance is less than the latitude of the place of observation. They do not set, but are always above the horizon of that place.

53. **The Right Ascension of the Meridian** of the place of observation is the siderial time at that place.

54. **The Siderial Time of Mean Noon** at any place is the right ascension of the mean sun at that time.

55. **In the General Spherical Triangle of Position,** P Z S, P is the Pole, Z is the Zenith, and S is the Star, or other celestial object. The side P Z is the meridian of the place of observation, P S is the meridian of the star, and Z S is in the vertical circle of the star. Z P S is the hour angle, P Z S is the azimuth, and P S Z the Parallactic Angle, of S.

56. **The Apparent Daily Path** of a star is the circumference of a circle with its center in the axis of the celestial sphere and its plane perpendicular to that axis. Except on the equator, these will be small circles of the sphere. Such is the small circle in Fig. 6 whose radius is C S.

57. **The Elongation** of a star occurs at the two places in its daily path where its verticle circle is tangent to that path. These places are not diametrically opposite but a little nearer the zenith than is the pole. The star moves in a vertical for a brief time while at an elongation, passing upward at its eastern elongation, and downward at its western elongation. These are the most favorable

positions for azimuth observations, besides stars not far from the pole have a slow apparent motion, allowing time for those with but little experience to make the observations. In the telescope of an ordinary engineer's transit a star near the pole will seem to pass directly along the vertical cross wire for a considerable time, if the instrument is kept pointed on the star. As the polar distance, P S, of the star is the spherical radius of its daily path, and as the plane of Z S is tangent to this daily path at an elongation, P S Z is a right angle and P Z S a right triangle to which the equations for spherical right triangles apply.

58. The few astronomical terms mentioned here are only sufficient for understanding what follows and for the limited use of the Ephemeris required herein. Those who may desire a knowledge of Astronomy, or a more complete understanding of the terms here used, should consult the standard works on Spherical and Practical Astronomy. For the use of engineers, one of the best is Practical Astronomy, by W. W. Campbell, of the Lick Observatory, published by the MacMillan Company of New York. A standard work for the use of astronomers is Spherical and Practical Astronomy, by Wm. Chauvenet.

NOTATION

59. Stars in the various constellations are designated by the letters of the Greek alphabet, by numbers, and by the notation of star catalogues. Many of the more prominent stars are named, as **Sirius, Polaris, Algenib, Regulus.** The same star may have a name, as **Polaris,** and another designation, as α **Ursae Minoris.** The first letters of the Greek alphabet are generally used to distinguish the stars in the order of their brightness.

GREEK ALPHABET

LETTERS	NAMES	LETTERS	NAMES	LETTERS	NAMES
Α, α,	Alpha	Ι, ι,	Iota	Ρ, ρ,	Ro
Β, β,	Beta	Κ, κ,	Kappa	Σ, σ, ς,	Sigma
Γ, γ,	Gamma	Λ, λ,	Lambda	Τ, τ,	Tau
Δ, δ,	Delta	Μ, μ,	Mu	Υ, υ,	Upsilon
Ε, ε,	Epsilon	Ν, ν,	Nu	Φ, φ, ϕ,	Phi
Ζ, ζ,	Zeta	Ξ, ξ,	Xi	Χ, χ,	Chi
Η, η,	Eta	Ο, ο,	Om~~r~~icron	Ψ, ψ,	Psi
Θ, θ,	Theta	Π, π,	Pi	Ω, ω,	Omega

ϕ = latitude of place of observation.

δ = declination. δ_0, δ_1, δ_2, etc., for particular stars, and δ_m the least declination that can be used.

α = right ascension. α_0, α_1, α_2, etc., for particular stars, α_m the least, and α_n the greatest value of α that can be used.

t = hour angle, with same subscripts as α. t_n = 6 hours.

h = altitude with similar subscripts.

T = astronomical time of any observation, that is, its time from noon, with similar subscripts.

ν_0 = "Siderial Time of Mean Noon" at Washington for the date of the observation, taken from the table headed "Solar Ephemeris" in the American Ephemeris. In case some other ephemeris is used, as that for Greenwich, or a Nautical Almanac, get from it the right ascension of the mean sun at mean noon for the date of the observation. To do this take the sun's apparent right ascension and the equation of time from the table giving the sun's coordinates at "Mean Noon." Add the equation of time to the sun's apparent right ascension if the table says "Added to Mean Time" or subtract it if the table says "Subtracted from Mean Time." The result will be ν_0.

A = azimuth of a star at its elongation, with the same subscripts as δ.

60. Some knowledge of the names and relative positions of stars is needed. For the latitude of the United States any of the small planispheres, such as Whitall's, or Harrington's, will do. For something more elaborate a star atlas such as Proctor's or Klein's, will be satisfactory. Plate II is a map of the stars of the north polar region for engineers' use.

61. There will also be needed a table of the Apparent Places of fixed stars, such as is given in the American Ephemeris, or some similar work. This book also furnishes some other data that may be needed by a beginner in preparing for the observations. After some experience no such preparation will be needed. One can observe at once without any previous calculations.

62. Observe the above notation and the conditions and instructions stated below. For reducing the data obtained by observing, the ordinary mathematical tables, such as Hussey, Jones, Searles, or Vega are used.

PREPARATIONS

63. The limiting values of δ, t, and a depend upon the limit of altitude which the instrument to be used is capable of observing. Examine the instrument to be used and ascertain the greatest altitude it will give comfortably. Try it with the telescope both sides up. In common field transits this is from 45° to 55°, after which the eye-piece comes in the way of the edge of the plate. This is h_n.

Then
$$\sin \delta_m = \frac{\sin \phi}{\sin h_n} \tag{11}$$

and
$$\cos t_m = \frac{\tan \phi}{\tan \delta_m} \tag{12}$$

If ϕ is not known take it from a map or guess at it, being sure to use a value a little too large.

By judgment assume T_m the earliest hour from noon when observations can begin on account of the daylight, and T_n the latest hour from noon at which it is desired to continue the observations.

Then for Eastern Elongations,

$$\begin{aligned} \alpha_m &= T_m + \nu_0 + t_m \\ \alpha_n &= T_n + \nu_0 + 6^h \end{aligned}$$

and for Western Elongations,

$$\begin{aligned} \alpha_m &= T_m + \nu_0 - 6^h \\ \alpha_n &= T_n + \nu_0 - t_m \end{aligned}$$

If α_m or α_n is more than 24 hrs., reject 24 hrs.

64. From the tables of Apparent Places of Fixed Stars choose the stars of proper magnitude whose declinations are greater than δ_m and whose right ascensions are between α_m and α_n for the eastern and western elongations, as the case may be. Find the times of the elongations of these stars, as follows:

Find the hour angle by

$$\cos t = \frac{\tan \phi}{\tan \delta} \tag{13}$$

and the times by $T = \alpha \pm t - \nu_0$. $+ t$ for western and $- t$ for eastern elongations. If α is not large enough to subtract t and ν_0 from, or if $\alpha + t$ is less than ν_0, add 24 hours. Having found the times, revise the list, setting down the times in the order of their succession with the names of the proper stars against them, their magnitudes, and a note of their elongations, E. or W. It may be well to add their declinations also to save looking them up again.

65. In a certain set of observations for March 10th, 1875, the instrument to be used was found to indicate an altitude of 50° with comfort. The latitude was thought to be something less than $43^\circ = \phi$.

$$\sin \delta_m = \frac{\sin 43^\circ}{\sin 50^\circ} \qquad \begin{array}{lr} \log \sin 43^\circ & 9.8338 \\ \text{`` `` } 50^\circ & \underline{9.8843} \\ \text{`` `` } \delta_m & 9.9495 \end{array}$$

$$\delta_m = 62^\circ\ 55'$$

$$\cos t_m = \frac{\tan \phi}{\tan \delta_m} = \frac{\tan 43^\circ}{\tan 62^\circ\ 55'} \qquad \begin{array}{lr} \log \tan\ 43^\circ & 9.9697 \\ \text{`` `` } 62^\circ\ 55' & \underline{0.2913} \\ \text{`` } \cos t_m & 9.6784 \end{array}$$

$$t_m = 61^\circ\ 31' = 4^h\ 6^m$$

The earliest hour work could begin was 7:30 P. M. Therefore $T_m = 7^h\ 30^m$. The latest hour it was desired to observe was $11^h = T_n$. ν_0, from the Solar Ephemeris Table for 1875-3-10 is about $23^h\ 12^m$.

$$\text{E. Elong.} \quad \begin{aligned} a_m &= 7^h\ 30^m + 23^h\ 12^m + 4^h\ 6^m &&= 34^h\ 48^m \text{ or } 10^h\ 48^m \\ a_n &= 11^h + 23^h\ 12^m + 6^h &&= 40^h\ 12^m \text{ or } 16^h\ 12^m \end{aligned}$$

$$\text{W. Elong.} \quad \begin{aligned} a_m &= 7^h\ 30^m + 23^h\ 12^m - 6^h &&= 24^h\ 42^m \text{ or } 0^h\ 42^m \\ a_n &= 11^h + 23^h\ 12^m - 4^h\ 6^m &&= 30^h\ 06^m \text{ or } 6^h\ 06^m \end{aligned}$$

66. The following list of stars was selected from the table in the American Ephemeris giving the Apparent Places of Fixed Stars. The declinations are greater than 62° 55′ and the magnitudes are found from the preceding table of Mean Places.

E. Elong.

Name	a	δ	Mag.	T, about
λ Draconis,	$11^h\ 24^m$	70° 01′ 09.″4	3.4	$7^h\ 31^m$
4 Draconis,	$12^h\ 06^m$	78° 18′ 30 ″1	5.4	$7^h\ 39^m$
κ Draconis,	$12^h\ 28^m$	70° 28′ 26.″4	3.4	$8^h\ 34^m$
32 Camelop,	$12^h\ 48^m$	84° 05′ 19.″3	5.4	$7^h\ 58^m$
α Draconis,	$14^h\ 01^m$	64° 58′ 02.″9	3.4	$10^h\ 32^m$
etc.	etc.	etc.	etc.	etc.

W. Elong.				
α Ursæ Minoris,	$1^h\ 11^m$	88° 37′ 47.″4	2.0	$7^h\ 55^m$
50 Cassiopeæ,	$1^h\ 53^m$	71° 49′ 07.″5	4.0	$7^h\ 30^m$
ι Cassiopeæ,	$2^h\ 19^m$	66° 50′ 34.″1	4.0	$7^h\ 33^m$
etc.	etc.	etc.	etc.	etc.

the times, T, being found by the equation as follows:

$$\cos.\ t = \frac{\tan\phi}{\tan\delta}$$

For λ Draconis

$$\begin{array}{ll} \delta = 70^\circ\ 01' & \alpha = 11^h\ 24^m \\ \phi = 43^\circ & \nu_0 = 23^h\ 12^m \end{array}$$

$$\begin{array}{llrl} \text{log tan } 43^\circ & 9.9697 & & \\ \text{`` `` } 70^\circ\ 01' & \underline{0\ 4393} & & \\ \text{`` cos } t & 9.5304 & t = 70^\circ\ 10' = & 4^h\ 41^m \\ & & & \underline{23^h\ 12^m} \\ & & & 27^h\ 53^m \\ & & 24^h + 11^h\ 24^m = & \underline{35^h\ 24^m} \\ & & T = & 7^h\ 31^m \quad \text{E. Elong.} \end{array}$$

For α Ursæ Minoris

$$\delta = 88^\circ\ 38' \quad \alpha = 1^h\ 11^m$$

$$\begin{array}{llrl} \text{log tan } 43^\circ & 9.9697 & & \\ \text{`` `` } 88^\circ\ 38' & \underline{1.6224} & & 1^h\ 11^m \\ \text{`` cos } t & 8.3473 & t = 88^\circ\ 43' = & 5^h\ 56^m \\ & & & \underline{24^h\ 00^m} \\ & & & 31^h\ 07^m \\ & & & \underline{23^h\ 12^m} \\ & & T = & 7^h\ 55^m \quad \text{W. Elong.} \end{array}$$

The revised list was then made out as follows:

T	Elong.	Mag.	Name	δ
7.30	W.	4.0	50 Cass.	71° 49′ 07.″5
7.31	E.	3.4	λ Draco.	70° 01′ 09.″4
7.33	W.	4.0	ι Cass.	66° 50′ 34.″1
7.39	E.	5.4	4 Draco.	78° 18′ 30.″1
etc.	etc.	etc.	etc.	etc.

67. It may be noticed that $h_{\prime\prime}$ must be considerably larger than ϕ in order for the instrument to be used at all. The work of preparing such a list can readily be abbreviated. With some experience a list may be written at once without figuring. When continually at work at the same place the list may be extended from day to day by simple inspection. The changes in the values of $a_{\prime\prime\prime}$ and $a_{\prime\prime}$ depend upon the change in the value of ν_0. The change in T is mainly due to the change in ν_0. The extreme detail in the above work is for a novice,—not as an example of practice. Western elongations are to be preferred for the smaller field instruments, because, if the star can be observed before it reaches its elongation it can be seen at its elongation. The data used being approximate, it may happen that a star at its eastern elongation, may pass a little too high for the instrument.

68. Provide some means of illuminating the cross wires so they can be seen at the same time the star is. Instruments of the more precise grades intended for night work, and mining instruments, are fitted with apparatus for this purpose. The common engineer's transit for field use generally is not. For such an instrument, tie a piece of unglazed white paper (common newspaper) over the objective by a string around the telescope tube back of the lens, with a hole in the paper centrally over the objective. This hole may be from ¼″ to ½″, according to the size of the lens and its aperture. Light shining upon the outside of this paper will render the cross wires visible. A lantern with a lens throwing light in one direction is best. The common lantern may be made use of by nearly surrounding it with something to shut off the light. The light must not shine in the observer's face. It must therefore shine obliquely from the side upon the paper over the objective. It is possible to observe without an assistant, but difficult. The paper over the objective is a protection to it. The light may be toned to any faintness within the telescope by withdrawing the lantern or changing the obliquity of the beam of light. The lantern will also be used to read the plate. It must not be held too near the plates or its heat may change the readings.

69. Choose some point of the line whose direction is sought, or reference line, from which a clear view of the stars to be observed may be obtained, for an instrument point. Fasten a lantern at some other point of the reference line to sight to. A common lantern about a mile away will do very well. If necessary to have it nearer, a fainter light may be made by covering part of the lantern globe. This light should be faint enough to look about like a faint star, so that no change of focus of the objective will be required.

OBSERVATION

70. In good season for beginning the observations, set up the instrument over its point of the reference line. Sight to the distant lantern on its point of the reference line. In time for the first star, illuminate the cross wires and set the line of sight upon the distant lantern. See that the spindle clamp is secure. Read the plate. Record the readings. Release the plate clamp. Focus the objective for the stars, if it needs it, for which purpose the moon, or a planet, is useful. Point the telescope at the star to be observed and bring it into its field of view. Illuminate the cross wires. Set the vertical wire, by means of the plate clamp and slow motion screw, on the star. Keep this wire on the star very carefully as it approaches its elongation, which may be known by the increased time taken by the star to cross the wire. Follow the star with the plate slow motion screw, keeping the vertical wire exactly on it till it appears to pass directly along the wire. Read the plate. Record the readings, marking them for the star observed. Release the plate clamp. Point the telescope to the distant lantern. Illuminate the cross wires and set the line of sight again upon this lantern by means of the plate clamp and slow motion screw. Read the plate. Record the readings. Compare them with those taken before observing the star. If they differ too much from them the observation must be rejected. The amount of this difference depends upon the instrument and the class of the work.

71. In time for another star, read the plate with the line of sight set again upon the reference line and record the readings. Release the plate clamp, point the telescope for the star, illuminate the cross wires, and observe it as the first one was observed. Record the plate readings for this star. Set the line of sight again upon the reference line by means of the plate clamp and slow motion screw, read the plate, record the readings, compare them with those before observing the star, and thus examine the position of the instrument as to stability. So do for every star observed.

72. Observe at least three stars in this way that give good observations, and as many more as may be desirable or possible.

REDUCTION

73. The observations are reduced as follows: Find, from the plate readings on the reference line and those for each star, the angles between the vertical ~~surface~~ ~~plane~~ circle of the reference line and those of the stars. From these angles find half the angle between the vertical circles of a pair of the stars. There are two cases: when the vertical circle of the reference line is btween the stars, and when it is not. In the first instance, take the half sum of the horizontal angles, in the second, their half difference. If the stars were at the same elongation, that is, if both E or both W, this will be half $(A_0 - A_1)$ in the equations below, if one is E and one W, this will be $\frac{1}{2}(A_0 + A_1)$. Take from the Ephemeris the apparent declinations, δ_0 and δ_1, of these stars for the date of the observation. By means of the following equations, using the proper one, compute $\frac{1}{2}(A_0 + A_1)$ or $\frac{1}{2}(A_0 - A_2)$, as the case may be:

(14)

$$\text{Tan } \tfrac{1}{2}(A_0 + A_1) = -\tan \tfrac{1}{2}(A_0 - A_1) \cot \tfrac{1}{2}(\delta_0 + \delta_1) \cot \tfrac{1}{2}(\delta_0 - \delta_1).$$

$$\text{Tan } \tfrac{1}{2}(A_0 - A_1) = -\tan \tfrac{1}{2}(A_0 + A_1) \tan \tfrac{1}{2}(\delta_0 + \delta_1) \tan \tfrac{1}{2}(\delta_0 - \delta_1).$$

Having thus obtained $\frac{1}{2}(A_0 + A_1)$ and $\frac{1}{2}(A_0 - A_1)$, find A_0 and A_1. These are the angles from the meridian to the vertical circles of the stars. Apply these values of A_0 and A_1 to the horizontal angles from the vertical circle of the reference line to the vertical circles of the stars, first above directed to be found, to get the horizontal angle from the meridian to the vertical circle of the reference line, or its direction. Notice how these values of A_0 and A_1 must be applied in order to give the quantity sought. The two values found for the direction of the line should be identical out to the last figure. If not it shows an error in figuring which must be found and corrected.

74. Three stars must be observed in order to get a check upon both the figures and the work. In combining the observations as above directed, those upon two stars at the same elongation having nearly the same δ must not be put in the same pair, because cot $\frac{1}{2}(\delta_0 - \delta_1)$ will be so large a multiplier for the errors of observation that the result will be found worthless. By this method the observations may have any interval of time between them. When reducing, use the apparent declinations of the stars at the time when they were observed. This fact permits all observations from the same reference line to be combined. This makes the final result more trustworthy. The observations are combined in pairs. This gives the greatest number of results from a given list of observations.

75. The latitude of the place of observation enters into the work only in such a loose manner as to enable one to use this method where the latitude is not known. After some experience it will not be needed at all. The observations may be made at any time and reduced at any other.

76. After the azimuths, A_0, A_1, etc., of the stars are found, the latitude may be found by:

$$\cos \phi = \frac{\cos \delta_0}{\sin A_0} = \frac{\cos \delta_1}{\sin A_1} = \frac{\cos \delta_2}{\sin A_2} = \text{etc.} \qquad (15)$$

This shows that the latitude may be found by the measurement of horizontal angles alone.

77. Of the above list of stars the following notes of observations upon three of them are taken as an example:

		Ver. A.	Ver. B.
Readings on Ref. Line,		15° 14′ 10″	195° 14″ 20″
" " λ Dr.,	E. Elong.,	132° 39′ 50″	312° 40′ 00″
" " α Urs. Min.,	W. "	103° 18′ 40″	283° 18′ 40″
" " κ Dr.,	E. "	132° 01′ 00″	312° 01′ 00″

The work of reduction is as follows:

	Ver. A.	Ver. B.
Angle from Ref. Line to λ Dr.,	117° 25′ 40″	117° 25′ 40″
" " " " " α Ur. Min.,	88° 04′ 30″	88° 04′ 20″
" " " " " κ Dr.,	116° 46′ 50″	116° 46′ 40″

	λ Dr. & α Ur. Min.	λ Dr. & κ Dr.	α Ur. Min. & κ Dr.
	117° 25′ 40″	117° 25′ 40″	116° 46′ 45″
	88° 04′ 25″	116° 46′ 45″	88° 04′ 25″
2.)	29° 21′ 15″	0° 38′ 55″	28° 42′ 20″
	14° 40′ 37″.5	0° 19′ 27″.5	14° 21′ 10″
	½ $(A_0 + A_1)$	½ $(A_0 - A_2)$	½ $(A_1 + A_2)$
	δ_0 70° 01′ 09″.4	δ_0 70° 01′ 09″.4	δ_1 88° 37′ 47″.4
	δ_1 88° 37′ 47″.4	δ_2 70° 28′ 26″.4	δ_2 70° 28′ 26″.4
2.)	158° 38′ 56″.8	140° 29′ 35″.8	159° 06′ 13″.8
$\frac{1}{2}(\delta_0 + \delta_1)$	79° 19′ 28″.4	$\frac{1}{2}(\delta_0+\delta_2)$ 70° 14′ 47″.9	$\frac{1}{2}(\delta_1+\delta_2)$ 79° 33′ 06″.9
2.)	—18° 36′ 38″.0	—0° 27′ 17″.0	18° 09′ 21″.0
$\frac{1}{2}(\delta_0-\delta_1)$	— 9° 18′ 19″.0	$\frac{1}{2}(\delta_0-\delta_2)$—0° 13′ 38″.5	$\frac{1}{2}(\delta_1-\delta_2)$ 9° 04′ 40″.5

λ Dr. & α Ur. Min.

Tan ½ $(\delta_0 + \delta_1)$	0.7246723
" ½ $(\delta_0 - \delta_1)$	9.2144487
" ½ $(A_0 + A_1)$	9.4182506
" ½ $(A_0 - A_1)$	9.3573916
½ $(A_0 - A_1)$	12° 49′ 40″
½ $(A_0 + A_1)$	14° 40′ 37″.5
λ Dr. A_0	27° 30′ 17″.5
α Ur. Min. A_1	1° 50′ 57″.5

Ang. to λ Dr.	117° 25′ 40″	Ang. to α Ur. Min.	88° 04′ 25″
A_0	27° 30′ 17″.5	A_1	1° 50′ 57″.5
Az. of Line,	89° 55′ 22″.5	Az. of Line,	89° 55′ 22″.5

λ Dr. & κ Dr.

cot ½ $(\delta_0 + \delta_2)$	9.5552189
" ½ $(\delta_0 - \delta_2)$	2.4014043
tan ½ $(A_0 - A_2)$	7.7528364
" ½ $(A_0 + A_2)$	9.7094596
½ $(A_0 + A_2)$	27° 07′ 21″.4
½ $(A_0 - A_2)$	0° 19′ 27″.5
A_0	27° 26′ 48″.9
A_2	26° 47′ 53″.9

Ang. to λ Dr.	117° 25′ 40″	Ang, to κ Dr.	116° 46′ 45″
A_0	27° 26′ 48″.9	A_2	26° 47′ 53″.9
Az. of Line	89° 58′ 51″.1	Az. of Line	89° 58′ 51″.1

α Ur. Min. & κ Dr.

tan ½ $(\delta_1 + \delta_2)$	0.7342345
" ½ $(\delta_1 - \delta_2)$	9.2035192
" ½ $(A_1 + A_2)$	9.4080330
—tan ½ $(A_1 - A_2)$	9.3457867
½ $(A_1 - A_2)$	— 12° 30′ 03″.1
½ $(A_1 + A_2)$	14° 21′ 10″
A_1	1° 51′ 06″.9
A_2	26° 51′ 13″.1

Ang. to α Ur. Min.	**88° 04′ 25″**	**Ang. to κ Dr.**	**116° 46′ 45″**
A_1	**1° 51′ 06″.9**	**A_2**	**26° 51′ 13″.1**
Az. of Line,	**89° 55′ 31″.9**	**Az. of Line,**	**89° 55′ 31″.9**

Reject the middle value of the azimuth of the line and the

Final Azimuth of Line = 89° 55′ 27″.2.

78. As the plate was numbered from 0° to 360° in right handed rotation and all angles read from the reference line round to the right, the result is the angle from the reference line reckoned to the right to the meridian. Since the instrument looked to the northern portion of the heavens this angle reads to the north point of the horizon, and the direction of this line is

N. 89° 55′ 27″.2 W.

79. The middle value of the azimuth of the line was rejected because of its manifest error, being some 3′ 25″ different from the others, which would exclude it from 10″ observations. It is thus in error because it was obtained from two stars at the same elongation whose declinations were nearly the same. The calculations were given to show the worthlessness of such a combination. A more trustworthy value of this angle from other sources is 89° 55′ 30″. By this one may perceive the value of the middle result. While some discrepancy appears in the original readings the final result is quite good. The changes of temperature which the parts of the instruments undergo in night work out of doors with lanterns held close to them works mischief with the readings, and frequent back readings on the reference line are necessary.

To get the latitude,

$\cos \delta_0$	9.5336500	$\cos \delta_2$	9.5240514
$\sin A_0$	9.6644764	$\sin A_2$	9.6548625
$\cos \phi$	9.8691736	$\cos \phi$	9.8691889
ϕ	42° 16′ 37″	ϕ	42° 16′ 29″.3

$\phi = 42° 16′ 33″.1$

More trustworthy value from other sources,

$$\phi = 42^\circ\ 16'\ 30''.$$

By this may be perceived the value of the method. Had the observations and reductions been extended somewhat, no doubt very close results could have been obtained.

BY MARKING THE DIRECTION OF THE STARS WITH STAKES

80. It may happen that one may be provided with a comparatively poor instrument yet need to determine the direction of a line with some precision. Put the instrument in as good working condition as possible and in as good adjustment as it can be. Observe a star with it as before directed but use no reference line and read no angles. After pointing to the star point downwards and set a stake marked for the star with a point carefully marked on it in line. Do the same for two or more stars, setting the stakes as far away as convenient yet near enough for the points on them to be well located. Reverse the telescope about both axes and level up the instrument anew after observing each star. Make notes showing the position of the telescope when pointing to each star. In combining the observations in pairs place together two stars for which the notes show the telescope to have been in its two different positions. By so doing the imperfections of the instrument will do less harm.

If I, in Fig. 8, shows the place of the instrument, C and D, the stakes for two stars, one at its E. elongation, and the other at its W. elongation, and $I\,K$ the meridian, make the following measurements carefully with a steel tape or similar tool,— $C\,D = d$; $I\,C = a$; and $I\,D = b$.

$$\text{Then } x - y = d\,\frac{a \cos \delta_0 - b \cos \delta_1}{a \cos \delta_0 + b \cos \delta_1} \qquad (16)$$

where $x = C\,K$, and $y = K\,D$, the segments of $C\,D$ formed by K, the point where the meridian from I intersects C D.

$x + y = d$. Find x and y and locate the point K with the instrument and steel tape. As a check on the figuring use

$$\frac{x}{y} = \frac{a \cos \delta_0}{b \cos \delta_1} \tag{17}$$

Since $A_0 = C\,I\,K$ and $A_1 = K\,I\,D$, $C\,I\,D = A_0 + A_1$ in Fig. 8, and $A_0 - A_1$ in Fig. 9.

Then Tan $\frac{1}{2}(A_0 - A_1) = -\tan \frac{1}{2}(A_0 + A_1) \tan \frac{1}{2}(\delta_0 + \delta_1) \tan \frac{1}{2}(\delta_0 - \delta_1)$ as before when A_0 and A_1 become known and

$$\cos \phi = \frac{\cos \delta_0}{\sin A_0} = \frac{\cos \delta_1}{\sin A_1} \text{ as before.} \tag{18}$$

Whence it appears that one may obtain the direction of the meridian and the latitude of the place without reading angles at all. A_0 and A_1 may be found **directly from** *the triangles* $C\,I\,K$ and $K\,I\,D$.

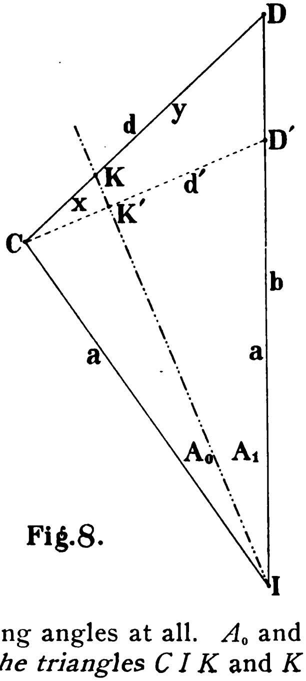

Fig. 8.

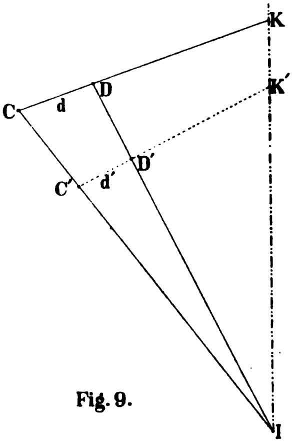

Fig. 9.